JHEP HYPER VERSION

Received: *December 23, 1997*, Accepted: *January 21, 1998*

Monstrous heterotic quantum mechanics

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Abstract: A compactification of the heterotic string is considered which describes a particle propagating in the symmetric space $\mathcal{M} = SO(9,1;Z)\backslash SO(9,1;R)/SO(9;R)$ in interaction with excited states whose dynamics is $1+1$ dimensional. The model appears to describe D-particle dynamics in type IIA string theory on $\mathcal{M}$.

Keywords: Superstrings and Heterotic Strings, String Duality, Superstring Vacua.

JHEP01(1998)012

Low dimensional compactifications of string/M-theory are of great interest for deciphering the underlying symmetries of the theory. The non-perturbative U-duality symmetry group becomes ever larger as the number of non-compact dimensions is reduced [1, 2, 3]. There are arguments (see *e.g.* [4, 5, 6, 7] for recent discussions and further references) that suggest that the symmetry group of two dimensional type II string theory (or M-theory on a nine-torus) is a discrete version of the affine group $\hat{E}_8$ (sometimes refered to as E_9), while in one non-compact dimension (time) the symmetry group is a discrete hyperbolic group known as E_{10}. At the same time, there are many conceptual problems with defining static vacua in low dimensional theories of gravity (see *e.g.* [8]).

Another aspect of low dimensional string theory is the observation of [9] further explored in [10] that low dimensional string theory may play an important role in describing in a unified way the worldvolume geometry of critical strings and other branes. This circle of ideas has not yet found its proper place in the emerging understanding of non-perturbative string theory, but it is believed that string theories with extended world-sheet SUSY may be of special significance in this regard [9, 10, 11].

In this note we discuss the properties of an unusual compactification of the standard heterotic string to one space-time dimension, which preserves all sixteen supercharges of the original ten dimensional theory.[1] Normally, such systems have a Narain moduli space of classical vacua. In $0+1$ dimensions, the low energy dynamics is instead described by a σ-model for the light fields whose target space is the classical moduli space. Clearly, to understand it we have to keep all the string modes that become light anywhere in moduli space. In different decompactification regions of the classical moduli space the spectrum of light states becomes continuous, revealing the higher dimensional nature of the theory. The example we will discuss has two special features:

(1) The low energy dynamics corresponds to quantum mechanics with a finite number of degrees of freedom everywhere in the moduli space of vacua. There is a finite energy gap that separates the low energy quantum mechanics from the field theoretic continuum.

(2) While the original heterotic string lives on $R\times T^9$, its space-time dynamics describes particle propagation on a different manifold $R\times\mathcal{M}$ which is curved and has a cusp.

We start with the standard compactification of the heterotic string on T^9. The left and right moving coordinates of the string $(\bar{x}^1,\cdots,\bar{x}^{25};x^1,\cdots,x^9)$ live on an even self-dual Lorentzian torus $\Gamma^{25,9}$; the 25×9 dimensional Narain moduli space of inequivalent tori

$$\mathcal{M}_{\rm Narain}=SO(25,9;Z)\backslash SO(25,9;R)/SO(25;R)\times SO(9;R)\ , \tag{1}$$

is parametrized by expectation values of the scalar fields

$$V_{ia}=\partial x^i\bar{\partial}\bar{x}^a e^{iEx^0}\ , \tag{2}$$

[1] This theory was incompletely discussed in [9].

where $i = 1, \cdots, 9$, $a = 1, \cdots, 25$, and $E = 0$ on shell. In the limit of decompactification to ten dimensions the torus takes the form,

$$\Gamma^{25,9} = \Gamma^{16} \oplus \Gamma^{1,1}_{R_1} \oplus \Gamma^{1,1}_{R_2} \oplus \cdots \oplus \Gamma^{1,1}_{R_9} \ , \tag{3}$$

where Γ^{16} is one of the two sixteen dimensional even self-dual Euclidean tori and the nine radii R_i labeling the $\Gamma^{1,1}$'s are taken to infinity. In this region of moduli space, the heterotic quantum mechanics approaches $9+1$ dimensional field theory.

However, there is an interesting projection of the compactified theory onto a subspace that defines a theory that has no continuum states and no field theory limit. This is the subspace of $\mathcal{M}_{\text{Narain}}$ in which the Lorentzian lattice takes the form,

$$\Gamma^{25,9} = \Gamma^{24} \oplus \Gamma^{1,9} \ , \tag{4}$$

where Γ^{24} is the Leech torus, which is the unique twenty-four dimensional even self-dual torus that has no vectors of length $\sqrt{2}$. Since Γ^{24} has no moduli all the moduli of $\Gamma^{25,9}$ are contained in the Lorentzian even self-dual torus, $\Gamma^{1,9}$, that has a nine-dimensional moduli space,

$$\mathcal{M} = SO(9,1;Z) \backslash SO(9,1;R)/SO(9;R) \ . \tag{5}$$

The projection of interest is obtained by performing an asymmetric orbifold twist on the left-moving Leech torus,

$$(\bar{x}^2, \bar{x}^3, \cdots, \bar{x}^{25}) \to -(\bar{x}^2, \bar{x}^3, \cdots, \bar{x}^{25}) \ . \tag{6}$$

The $c = 24$ left-moving CFT corresponding to (6) is the Monster module [12, 13]. The torus partition sum of the resulting heterotic vacuum is

$$Z(\tau, \bar{\tau}) = \frac{1}{2}\left[\left(\frac{\theta_3(\tau)}{\eta(\tau)}\right)^4 - \left(\frac{\theta_4(\tau)}{\eta(\tau)}\right)^4 - \left(\frac{\theta_2(\tau)}{\eta(\tau)}\right)^4\right] J(\bar{\tau}) \frac{\sqrt{\tau_2}}{\eta(\tau)^8} \sum_{(p_R, p_L) \in \Gamma^{9,1}} q^{\frac{1}{2}p_R^2} \bar{q}^{\frac{1}{2}p_L^2} \ , \tag{7}$$

where θ_i are Jacobi θ functions, J is the unique meromorphic modular invariant with vanishing constant term,

$$J(\bar{\tau}) = \frac{1}{\bar{q}} + 0 + C\bar{q} + \cdots \ , \tag{8}$$

and $q = \exp(2\pi i \tau)$. The fact that the constant term in (8) vanishes means that there are no dimension one operators in the monster module CFT. Furthermore, it is clear from (7) that the vacuum preserves the same sixteen supercharges as the original heterotic model on T^9 and the classical moduli space of vacua is $\mathcal{M}$ (5).

The light degrees of freedom of the theory are as follows. At generic points in $\mathcal{M}$ only the nine moduli fields Φ_i $(i = 1, \cdots, 9)$, with vertex operators,

$$V_i = \partial x^i \bar{\partial} \bar{x}^1 e^{iEt} \ , \tag{9}$$

and their sixteen fermionic partners have zero energy on shell. As we approach the point in $\mathcal{M}$ where

$$\Gamma^{9,1} = \Gamma^8 \oplus \Gamma^{1,1}_{R=\sqrt{2}} \ , \tag{10}$$

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where Γ^8 is the E_8 torus and $\Gamma^{1,1}_{R=\sqrt{2}}$ corresponds to a compactification on a circle of radius $R = \sqrt{2}$, additional light states appear from vectors with $p_L^2 = 2$, $p_R = 0$ in (7). These states $\Phi_i^{\pm}$ are described by the vertex operators

$$V_i^{\pm} = \partial x^i e^{\pm i\sqrt{2}\bar{x}^1} e^{iEt} \tag{11}$$

and, together with $\Phi_i \equiv \Phi_i^3$, fill out an adjoint multiplet Φ_i^a $(a = 3, \pm;\ i = 1, \cdots, 9)$ of the $SU(2)$ gauge symmetry generated by $\bar{\partial}\bar{x}^1$, $\exp(\pm i\sqrt{2}\bar{x}^1)$.

The low energy Lagrangian describing $\Phi_i^a(t)$ locally near a degeneration point (10) is the dimensional reduction of $N = 1$ SYM with gauge group $SU(2)$ in $9+1$ dimensions to $0+1$ dimensions. Globally, the manifold $\mathcal{M}$ is curved and has a cusp corresponding to (10). Obviously, there are images of this cusp obtained by acting with $SO(9,1;Z)$. However, there are no inequivalent cusps, such as ones at which a larger number of states become massless simultaneously. The low energy Lagrangian for the modes Φ (9) generically has the form,

$$\mathcal{L} = G_{ij}(\Phi)\dot{\Phi}^i\dot{\Phi}^j \ , \tag{12}$$

where G_{ij} is the Zamolodchikov metric on the symmetric space (5). This can be verified by standard techniques [14]. Near the cusp one must add the degrees of freedom (11) to obtain a non-singular description.

The system in question has a large number of 1/2 BPS states obtained as in [15] by putting the right-movers in their ground state and allowing arbitrary excitations (consistent with the projection (6)) of the left-movers. The level matching condition is

$$1 + \frac{1}{2}p_R^2 = \frac{1}{2}p_L^2 + N_L \tag{13}$$

and the energy of the corresponding states is

$$E = |\vec{p}_R| \ . \tag{14}$$

Restricting to the region of moduli space in which $\Gamma^{9,1} = \Gamma^8 \oplus \Gamma^{1,1}_R$ and taking the decompactification limit $R \to \infty$ gives $|\vec{p}_R|^2 = k^2 + |\vec{p}_{E_8}|^2$, where k is the continuous momentum in the non-compact x^1 direction and $\vec{p}_{E_8} \in \Gamma^8$. We see, therefore, that these BPS states form a continuum as $R \to \infty$, *i.e.* they are described by a $1+1$ dimensional field theory. However, they are separated from the low energy quantum mechanics by a gap. States with $\vec{p}_{E_8} \neq 0$ have energy $\geq |\vec{p}_{E_8}|$. States with $\vec{p}_{E_8} = 0$ cannot be massless since the monster CFT does not have dimension one operators. We conclude that the two dimensional field theoretic continuum is separated from the low energy quantum mechanics by an energy gap of order one in string units.

In view of string duality one can ask whether the vacuum described above has an alternative description. A natural candidate is a 'compactification' of type IIA string theory on the manifold $\mathcal{M}$ (5) in the presence of a D-particle. When the D-particle approaches the cusp of $\mathcal{M}$ (10), states corresponding to fundamental strings stretched between the particle and its image with respect to the Z_2 fixed point become light and enhance the $U(1)$ quantum mechanics to $SU(2)$.

Such a picture would provide a natural explanation of the $SU(2)$ quantum mechanics discussed above, as well as of the energy gap to the excited states. It would be an interesting example of consistent D-particle propagation on a curved space.

More work is necessary to establish whether type IIA strings can be consistently formulated on $\mathcal{M} \times R$ in the presence of a D-particle. It is possible that one will also need to place an orientifold at the cusp of $\mathcal{M}$ in order to satisfy the equations of motion. If this is the case, the charge of the orientifold plane would have to be positive to get an $Sp(1) \simeq SU(2)$ projection on a nearby D-particle (negative charge would instead give rise to $SO(2)$). It would be interesting to clarify this issue and show that such a vacuum has the right amount of SUSY.

The heterotic vacuum we have discussed seems also to be related to the supersymmetric $(2,1)$ heterotic string vacuum studied in [10]. In particular, the two have the same amount of supersymmetry and moduli space of vacua $\mathcal{M}$ (5). Thus, a better understanding of this theory may provide useful insights for the program of [9, 10].

Acknowledgments

We thank E. Martinec for useful discussions and the Aspen Center for Physics for hospitality. The work of D. K. is supported in part by a DOE OJI grant.

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